# Homework #1

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| **EE 242**  **Spring 2025** | **Yehoshua Luna**  **2322458** |

## Problem #1

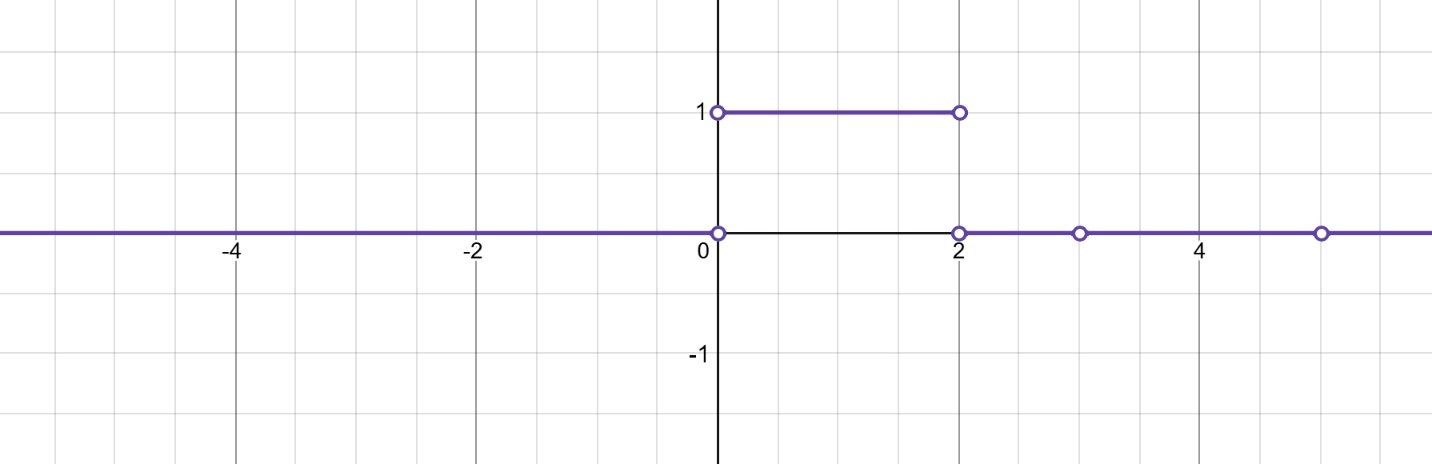
1. is defined for alland is therefore non-causal. We can also determine that it is neither even nor odd, since and . However, behaves periodically since it expands to , and these are both periodic functions with the same frequency. Lastly, because is periodic, defined for all , and has nonzero values, it cannot be an energy signal. Integration further backs this: , but . Therefore, it is a power signal.
2. is defined for all (though it is zero for those values) and is therefore non-causal. We also can see that it is neither even nor odd, since and . This function is not periodic because there is no value *T* such that for . Lastly, since is not periodic and grows unbounded for , it is neither an energy nor a power signal. The integration again shows this: and .

## Problem #2

The first term of is , which we know is an even function, and the second term is , which is an odd function. The last term can be expanded to using the identity , which is also an odd function. is a power signal because it is defined for all , periodic, and has nonzero values. It has a fundamental period of , so the power of is .

## Problem #3

We know that for , for , and . Now we are ready to make our graph of , which can be seen below. From this graph, we can also see that . Our result is validated by the simple fact that , which yields the following calculation: .



## Problem #4

From the question description, we have and want to determine the condition for such that is periodic. Combining and into a new frequency gives , so This discrete function is only periodic when is rational.

## Problem #5

1. From the complex Fourier Spectrum, we can see harmonics at , , , and . The largest fundamental frequency that can equal all of these values via some integer scaler is , so our fundamental period must therefore be .
2. All of the coefficients for the frequencies shown in the complex Fourier Spectrum are nonzero. It then follows that , , , , and . We can then express as the following sum: . This can be further simplified to .

## Problem #6

We must have and because is real and odd. This means that our expression for can be condensed to , since for all . Solving for requires some calculus: . Therefore, our final expression for the function is .

## Problem #7

It can easily be shown that is periodic through simple algebraic expansion: . This logic works because is periodic, thus is also periodic.

Finding an expression for the Fourier Series coefficients of is a bit trickier. We know that , therefore and . is simply a combination of both expressions, so we can add them together, giving . Combining and rearranging both terms then reduces the expression to a sum of coefficients . These coefficients can then be simplified using , thus . Now it becomes obvious that .

## Problem #8

We can express the Fourier Transform function in terms of sine and cosine instead of polar form, which gives . Cosine is even, and sine is odd. Furthermore, we know that all the cosine terms of a Fourier Series or Transform are zero when operating on an odd function, and all the sine terms are zero when operating on an even function. In other words, an even function can never have an odd function in its decomposition, and an odd function never has an even function in its decomposition. So if , then we know that the Fourier Transform of will consist of the only the cosine term, and will consist of just the sine term. But by definition, the cosine term is real, and the sine term is imaginary! Therefore, we have:

Given the previous logic, it then follows that and .

## Problem #9

1. . Each integral can now be solved after splitting:

So .

1. . We again can then just split the integral and solve:

So .

## Problem #10

1. . Taking the inverse Fourier Transform then gives:

## Problem #11

1. , so . This then gives .
2. , and , so .